SUPERMATH UNIT

Comparing Decimals

GOAL(S) OF THE UNIT: To learn how to determine the relative size of decimals.

Duration	3 days
Approach	Links size of decimals to addresses where the addresses get larger as you go uptown (i.e., move to the right in the 'hood'. The hood provides a mental model for the inside of a number line that enables students to develop an intuitive sense of place value and the size of decimals.
Supermath software	Where in the Hood is Carmen Decimal
Followup practice	Comparing decimals, interpreting decimals

Materials: A tape recording of a simulated newscast announcing the escape of Carmen SanDecimal from prison and last seen heading towards the hood near ???

—*where ??? is the name of your school*— school. Have a warning that Carmen is armed and dangerous.

A tape recording with a few seconds of silence and then a police siren or a police whistle.
Posters.

Pedagogical Approach:

The Carmen SanDecimal program is designed to provide a mental model from which students can derive their own rules for determining the relative size of decimals. The program is designed to be a metaphor for what the inside of a number line is like.

After the first day of this unit try to get students to talk about decimal values using the words that describe place values rather than just naming the digits of the number. For example: Students should say 473 thousandths for the number .473— Do NOT say 'point 4-7-3'. Care on your part to always use the place value vocabulary will help the students acquire a better understanding of place value.

In lesson #1 students at the Sergeant level need to figure out how to create an address for the alley between the buildings. Do not tell them how to do it. Let them experiment and figure it out.

Constantly engage students in discussion for their *general* impressions as to how decimals are formed in terms of the relationship of the structure of the digits between decimals of different sizes and place values.

(E.g., if students observe that .235 is bigger than .21, by the end of the unit they should be able to explain that both in terms of the program, i.e., .235 is an address to the right because it would be a mouse hole between the garbage cans of .22 and .23, while .21 would be a garbage can to the left (hence smaller), or by adding a zero so that it is .210, that would be a mouse hole between the garbage cans of .21 and .22. The students should also be able to use a math rule such that: When I move to the right of the decimal place, the hundredths place indicates that .235 is bigger since it has a 3 and the other only has a 1.

The third lesson is where you want students to develop, and transition to the use of, math rules for determining the relative size of decimals.

IDEA FORMULATION AND DEVELOPMENT

UNIT TITLE: COMPARING DECIMALS

SOFTWARE: WHERE IN THE HOOD IS CARMEN SANDECIMAL?

Pre-class setup: Load all student computers with 'WHERE IN THE HOOD IS CARMEN SANDECIMAL?' set to the Sergeant level.

Materials: A tape recording of a simulated newscast announcing the escape of Carmen SanDecimal from prison and last seen heading towards the hood near ??? —*where ??? is the name of your school*— school. Have a warning that Carmen is armed and dangerous.

Lesson #: 1

PRE-TEACHING ACTIVITIES (Demonstrations, linkages, discussion of ideas):

Today we are going to figure out how to compare decimals. This will be fun since decimals have lots of personality. Now most decimals are honest, round, and cuddly. But every once in a while a decimal goes bad and becomes crooked.

The worst of the crooked decimals is the master criminal, Carmen SanDecimal. What do you think a crooked decimal looks like? (Entertain student responses, e.g., a decimal with a scar, a decimal that looks like a square, a jagged decimal, etc.) If time permits let them draw what an evil decimal would look like.

Continue by saying:

I hope that we never meet a crooked decimal face to face. Turn on your tape recording announcing the escape of Carmen. Put on a worried look, and say:

Oh no! The hood is so big and Carmen is so crooked that the police are going to need help. Let's go to the hood and help them try and capture Carmen. Then say:

The honest, round decimals in the hood will help you and the police catch Carmen. They will give you clues as to the general area where the crooked decimal is hiding out. In addition, keep in mind that as you go through the hood, the decimals get larger as you move to the right. As you go to the right you are going uptown so the addresses get larger.

Write down the number of moves you take for each time you try to catch Carmen the evil decimal. Point to the chart in front of the room, and say: **Decimal detectives, to your computers.**

»» As students work at the computer, make sure that they record how many moves it took them to find Carmen.

Keep in mind that the post-teaching is critical to enable them to do the homework. ««

POST-TEACHING:

What was the trick to finding addresses between two buildings or garbage cans with consecutive numbers and the same number of places? (Add another decimal place to the address after the last digit of the smaller address.)

Since the addresses get larger as you move to the right, and the addresses are in the form of decimals, you can use the hood to help us figure out which decimals are larger. What would be a good strategy for using this game as a way of telling whether one decimal is larger than another? (Think of both as addresses, and see which direction you would have to move in to get from one to another.) Continue by saying:

Let's see if this strategy works. Think of the following decimals as addresses. Write the following on the board:

1.54, 1.5, 1.6, and 1.48

Think for a minute and see if you use the game to figure out the order of these decimals, from the smallest to the largest, and explain why you made the choices you did in terms of the position of addresses in the game. Have students answer by using examples of where the addresses would be on the screen, e.g., building or alley, to the right or left of each other, etc. (1.48, 1.5, 1.54, 1.6)

The homework for tonight contains practice in finding decimals in-between other decimals.

Hand out a copy of the homework to each student.

HOMEWORK

Think of what you learned from chasing Carmen, and write a decimal number that is between the values shown. Use as few places as possible to write your number.

1) .4 ____ .5

2) 1.7 ____ 1.8

3) .12 ____ .13

4) 8.45 ____ 8.46

5) 23.04 ____ 23.05

6) .07 ____ .08

7) 100 ____ 101

8) .497 ____ .498

9) .9043 ____ .904

10) Give an example of a three place decimal between .3 and .4_____

11) What is a possible two place decimal between 2 and 2.1? (Hint: Think about what 2 means as a one place decimal)

_____

IDEA FORMULATION AND DEVELOPMENT

UNIT TITLE: COMPARING DECIMALS

SOFTWARE: WHERE IN THE HOOD IS CARMEN SANDECIMAL

Pre-class setup: Load all student computers with 'WHERE IN THE HOOD IS CARMEN SANDECIMAL' by double clicking CARMEN SANDECIMAL icon and selecting the 'Sergeant' level from the opening screen. Hide a tape recorder with about 15 seconds of silence followed by a recorded police siren in your desk or some other location where it will be out of students view.

Materials: Tape recorder with a tape that has 15-30 seconds of silence followed by 15-30 seconds of police sirens.

Lesson #: 2

PRE-TEACHING ACTIVITIES (Demonstrations, linkages, discussion of ideas):

Answers to the homework

There are many possible answers to each question. The following show the range of possible answers. Accept any answer in the interval.

1) .41-.49	5) 23.041-23.049	9) .9041 or .9042
2) 1.71-1.79	6) .071-.079	10) Anything between .301 and .399
3) .121-.129	7) 100.1-100.9	11) Anything between 2.01 and 2.09
4) 8.451-8.459	8) .4971-.4979	

Secretly start the tape and gain the students attention as if you are about to start class. When the 15 seconds of silence is over, and the siren begins, listen with a puzzled expression on your face for a few moments and then say:

Oh no! That siren is bad news. What do you suppose has happened? Entertain student response. Then walk quickly to the door and pretend to be receiving a message from someone in the hallway. Return quickly to the classroom and say:

This is terrible news. Carmen San Decimal has escaped from prison again along with her group of crooked decimals. They have probably learned some new tricks in prison that will make them even more difficult to capture than the last time so we are going to have to be smarter and faster. Then say:

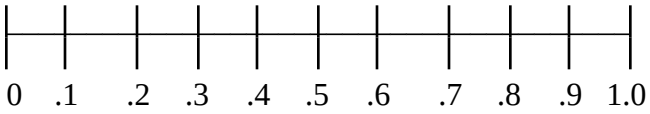
By the way, yesterday we compared decimals with the same number of decimal places. Let's get more complicated. Which decimal do you think is larger?

Write 2.4 and 2.3999 on the board.

Let students speculate, perhaps take a vote. DO NOT TELL THEM THE ANSWER!

Well I am not going to tell you the answer now because detectives should be able to figure it out on their own. Continue by saying:

Let's briefly review what we did as rookie decimal detectives and see if we can bring in some of our mathematical knowledge to help us out today. Draw the following section of a number line on the board and say:



Mathematicians call a diagram such as this a 'NUMBER LINE', but we know that it really is a map of the block on the hood where we started searching for Carmen the last time. Point to the number line and ask: **What do the single place value decimals under the vertical lines represent on this map?** (Building addresses.) **How many spaces or alleys do the building addresses divide the entire block into?** (10) **Therefore each space is one-tenth the distance of the block and that's why one place decimals are called tenths.** Then ask:

What do we know about Carmen's location if it turns out that she is between two consecutive tenths? (She must have gone down the alley between the 2 buildings.) **If the garbage cans with 2 decimal place addresses make ten spaces in each alley, how many garbage can spaces are there in the whole hood?** (100) **That is why two place decimals are hundredths. Using the same logic why would 3 decimal places be thousandths?** (Because there would be 1000 spaces between mouse holes.) **Let's quickly review. What are the names of the decimals we have talked about?** Students will probably answer tenths, hundredths, and thousandths. If they do, say: **Good, but you have forgotten the name of one decimal.** (Carmen SanDecimal.) Continue by saying:

CHALLENGE ACTIVITY:

Carmen is even trickier now so you need to find her more quickly by refining your search strategies. If you are real good you should always be able to locate the crooked decimal in less than 13 moves.

In addition, as you search for Carmen, I want you to think about the conditions under which a decimal is larger than another based on the number of decimal places and the value of the number in each decimal place. Watch the addresses on the garbage cans for clues as to how decimals are related to each other. Then say:

Go to your squad cars and see how many of the crooked decimals you can put back behind bars by the end of the day. Keep track of how many moves it takes for each game you play.

»» Cruise as students play the game to make sure they are formulating strategies for locating decimals between consecutive values and that they are writing down the number of moves they make in each game. ««

POST-TEACHING: With about 10 minutes left, stop the group and ask:

How many of the crooked decimals were you able to put behind bars? Entertain student response. **How many times did you capture Carmen with less than 13 moves?** Entertain student responses. Then ask:

Keeping in mind that as you move to the right addresses and decimals become larger, what would be a good strategy for using this game as a way of telling whether one decimal is larger than another? (Think of both as addresses, and see which direction you have to move in to get from one to another.)

Good idea! In terms of moving in the hood, which decimal would be larger 2.03 or 2.035? (2.035 because that would be in the alley between 2.03 and 2.04, or to the right of 2.03.) **Now let's go back to the problem on the board from before. Using the same logic, which is bigger, 2.4 or 2.3999?** (2.4, because 2.399 would be in the alley between 2.3 and 2.4, so 2.4 is to the right.)

So far you have used the simulation to tell whether decimals you have experienced in the game are larger or smaller than others. Tonight I want you to go imagine what the game would be like for even more pinpoint decimals you have not experienced. I want you to think about how you would design the game to work if when you got to the mouse holes you still had not found the crooked decimal.

HOMEWORK

1. For each of the following, explain your answer in terms of how decimals in the hood are organized in the Carmen San Decimal game. If it helps, draw a picture of where the decimal would be in the hood.

   a) Clem and Rosetta each caught a frog to enter in a frog-jumping contest. Clem's frog's biggest jump is 13.051 inches. Rosetta's frog's biggest jump is 13.1 inches. Whose frog will probably do better in the contest?

   b) Diane has a dress-making shop that uses very fine needles for sewing delicate fabric. She likes to use the smallest needles she can get. Is she better off with needles that have a diameter of .003 inches or .029 inches?

2. Use your experience with capturing Carmen SanDecimal to determine which of the decimals in the pair is the largest:

   a. 12.5 11.7

   b. 10.3 10.29

   c. .359 .362

   d. 6.08 6.1

   e. .222 .099

   f. 5.81 5.78

   g. .009 .01

   h. 3.1 2.9

IMAGINATION BUSTER problem:

3. Suppose you still had not found Carmen in one of the mouse holes, but you had found that she was between 2 of the mouse holes, write a short description of what you imagine would happen when you went into the wall between the two mouse holes. a) Describe what things would look like, b) describe what the addresses would be like, and c) write two decimals that you could compare at that level.

IDEA FORMULATION AND DEVELOPMENT

UNIT TITLE: COMPARING DECIMALS

SOFTWARE: WHERE IN THE HOOD IS CARMEN SANDECIMAL

Pre-class setup: Load all student computers with 'WHERE IN THE HOOD IS CARMEN SANDECIMAL' by double clicking CARMEN SANDECIMAL icon and selecting the 'Detectives Duet' level from the opening screen.

The following charts:

STRATEGY FOR SOLVING PROBLEMS

Look for patterns by examining several related problems

NAMES FOR DECIMAL PLACES

1st Decimal Place: Tenths
2nd Decimal Place: Hundredths
3rd Decimal Place:Thousandths
4th Decimal Place:Ten Thousandths
5th Decimal Place:Hundred Thousandths
6th Decimal Place: Millionths

Materials: Copies of the homework at the end of this lesson.

Lesson #: 3

PRE-TEACHING ACTIVITIES (Demonstrations, linkages, discussion of ideas):

HOMEWORK ANSWERS

1. a) Rosetta's frog, because 13.1 is *bigger* than 13.051
   b) .003, because that is *smaller* than .029

2. a) 12.5 b) 10.3 c) .362 d) 6.1 e) .222 f) 5.81 g) .01 h) 3.1

3. a) Students can come up with a variety of answers such as nine termite holes. There have to be nine of some tiny thing in the wall, b) the address of each thing would be a four place decimal, and c) an example would be to compare 2.1234 and 2.1236

As we saw in the answer to problem #1b, sometimes smaller decimals are better. What is something else in life besides sewing needles where smaller can be better? (e.g., portable telephone, hearing aid, how long it takes to run a race, how long you have to do homework, etc.)

BRAINSTORM:

Point to the results on the board from homework problem #2, and say:

I want you to use your experience in comparing decimals to develop a math rule that you could use to explain to someone who had never seen the Carmen SanDecimal game how to tell which of two decimals is larger. Point to the chart in front of the room and say:

One strategy for finding math rules is to look for patterns. Look for a pattern in the answers to the homework problems on the board that can be a math rule for explaining how to tell which decimal is the largest that does not depend on knowing Carmen SanDecimal.

Develop a one or two sentence math rule for deciding which of two decimals is larger than another, and a math rule for deciding whether a mixed decimal number such as 2.35 is bigger than another. Make the rules as simple as possible. Write out the rules and then I will see which team has the simplest correct rules.

Answer: The rules should be something like:

a) Start from the left and compare one decimal place at a time, and the first decimal place you come to that one has a larger number, that is the largest decimal.

b) The mixed decimal with the largest whole number is the largest. If they both have the same whole number, use the first rule to figure out which is largest.

This is great. You are able to go from your experience in playing a game to develop your own math rules. Based strictly on your math rule, do you think that 2.035 is larger or smaller than 2.03462345? (2.035)

In comparing decimals the biggest problem is to compare decimals with different numbers of decimal places. A simple trick is to add zeros to the right of the decimal with the fewest number of decimal places so that the number of places matches up. The same problem now becomes:

2.03500000 and 2.03462345

The reason why I asked you to develop a math rule is because decimals can get real long, and once we go beyond 3 or 4 decimal places it becomes difficult to visualize the decimals. If you were going to develop the Carmen game to have addresses that were 4 decimal places, how would you design it? (e.g., 9 ant holes in-between the mouse holes.)

At the same time, you developed a powerful math that will work for any size decimal. Continue by saying:

Now that you used patterns to figure out the math rule, let's think about why the math rule works. What happened to the things that had addresses and the spaces between addresses in the game as you used more and more decimal places? (They got smaller and smaller.)

If you wanted to exactly locate where a speck of dust was in the hood or on the number line, would you usually use one decimal place or many to indicate its address—Why? (Many decimal places because such numbers describe tiny spaces.)

As a result, the more decimal places you use when measuring or calculating the more exact or accurate the result will be. When someone is concerned about being very precise, they use decimals with many places. Decimals with many places are sometimes called 'Pinpoint' accuracy. Can you guess why? (Because the size of each space will be as small as a pinpoint if you use a lot of decimal places.)

Indeed, as you move to the right, the relative size of the type of address each decimal place represents is so much smaller than the one before it, that as soon as you come to the first decimal place that has a larger number, that decimal will be larger no matter how big the number is in any other decimal place to the right or how many other places there are to the right since they will only represent a much tinier a space or value. In other words, one building (1 decimal place) is much bigger than 1,000 mouse holes. Then point to the chart of decimal place values, and say:

Let us see if you can apply the principle that values get so much tinier every time you move one decimal place to the right. This chart has the name of the first six decimal places. As you move down the list each of the decimal place values is tinier than the one before. So what is bigger— 10 10 thousandths or 5000 millionths? (10 10 thousandths)

CHALLENGE ACTIVITY:

You did so well yesterday at finding the crooked decimals that you have all been promoted to Decimal Detectives. Then yawn and say: **But you cleaned up the streets so well yesterday that no crimes have been reported for you to investigate and you have nothing to do. Go to your computers and see what detectives do when they are bored.**

NUMBER OF STUDENTS PER COMPUTER: Two-three. If you are using 3 to a computer, since only 2 can play at a time, have the students alternate playing.

POST-TEACHING:

Congratulations to all of you on your fine detective work, or should I say 'eat'ective work. Write the word eatective on the board. Then say:

Suppose you forget later on the rules for decimals and place values that you were taught. How could you use the Carmen game to help you with math later on? (By remembering how to play the game we can remember how place value works.) **So even though Carmen is a computer simulation, by remembering how you played it, it can become a mental simulation later on for remembering the rules for comparing decimals by playing the game in your head. At the same time, you have also figured out a math rule on your own and you understand the reason for it. You are now real mathematicians. You should have no trouble applying your talents to the situations on the homework tonight.** Hand out a copy of the homework to each student.

HOMEWORK

1. To decode a secret message, write each of the following letters in the order of the smallest to the largest decimal.

_____

A .132F .8531	H .37009	I .58	M .0504
N .987	S .642	T .25	U .901

2. Ski races are usually won by very small differences. Suppose that Smiley raced down the hill in 15.1346 seconds and Jose in 15.1299 seconds. Who won the race?

_____

3. Glenda makes a pie that contains 56.009 grams of fat per serving. Bobby makes the same kind of pie using a different brand of shortening that contains 56.0082 grams of fat per serving. If you are on a low fat diet, which pie should you choose?

4. Write the following decimals in order from the smallest to the largest:

3.14 2.99567 3.09 3.0981 3.134987 3.135 3.0982

ANSWERS TO HOMEWORK:

1) MATH IS FUN 2) Jose 3) Bobby, because 56.0082 means a smaller amount of fat 4) 2.99567 3.09 3.0981 3.0982 3.134987 3.135 3.14